Introduction to the course

Jess Kunke

MATH/STAT 394: Probability I (Summer 2022 A-term)

Outline

Survey results

What is randomness, uncertainty, noise?

Course overview

Some notes on mathematical thinking

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Upcoming feedback:

- Midterm feedback (anonymous poll on pace, difficulty, general feedback)
- Ongoing anonymous feedback (see Canvas)

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- What is "noise"? Are the words randomness, uncertainty and noise interchangeable? (We will often use them that way)

Probability

We often want to study things that involve randomness, uncertainty, or noise

- Stock markets
- Weather
- Earthquakes
- Population health
- ...

One valuable approach: build mathematical models of the system or problem

We use probability to quantify the randomness/uncertainty/noise in these models

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Probability is used to **measure** how likely an **uncertain** event is to occur

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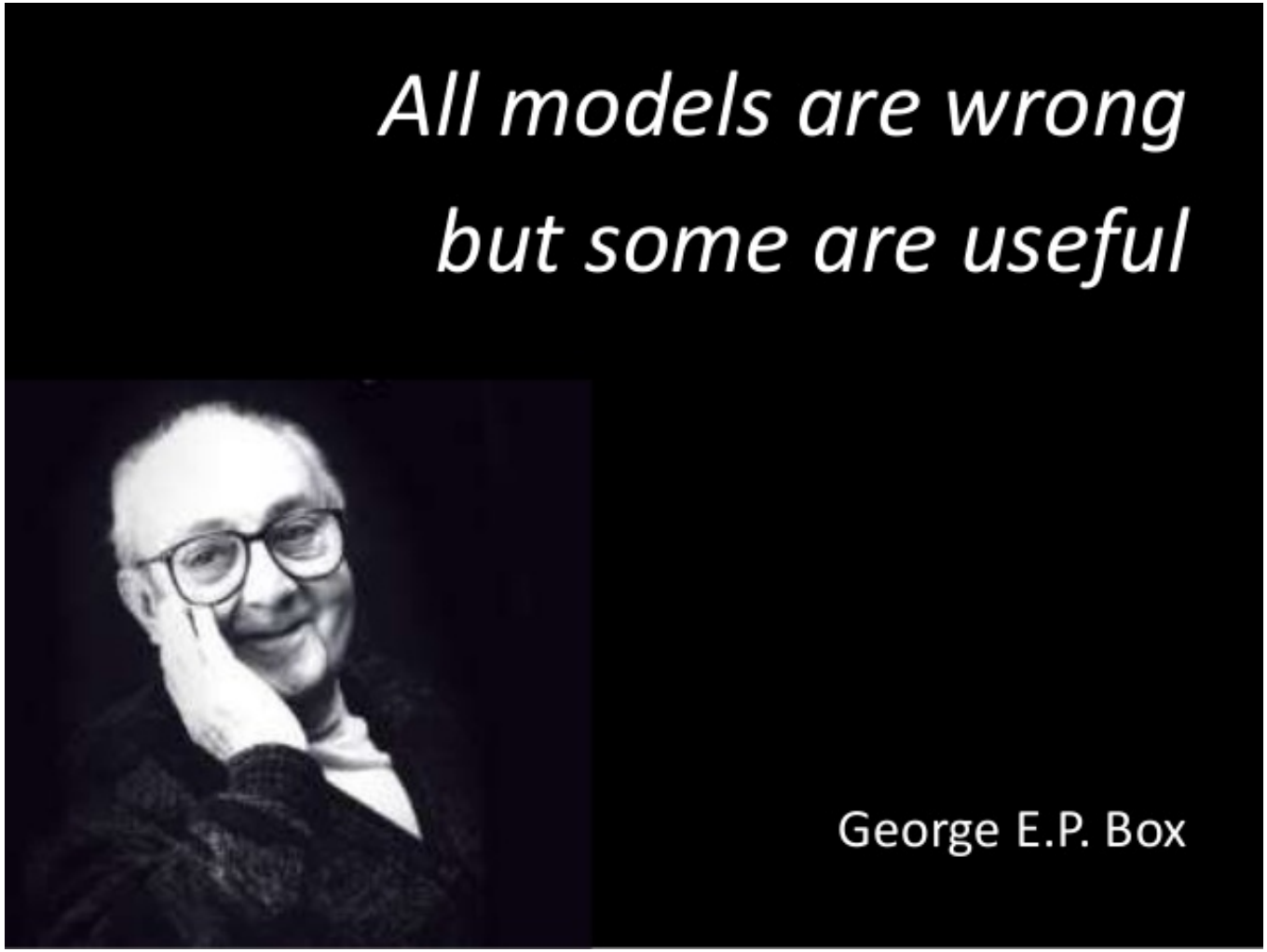
What about statistics?

- Probability theory develops the theoretical tools to frame randomness
- Statistics uses this framework to account for uncertainty when analyzing data or modeling a problem

A note about models



Outline

This course

Content of the course

- Review counting rules for finite sets
  - Measuring finite sets = counting how many elements they have
- Probability definitions
  - Framework, axioms, basic rules to compute probabilities
  - Classical examples: rolling a die, flipping a coin, sampling from an urn
  - Conditional probabilities, independence
- Random variables
  - Definition, classical examples
  - Discrete and continuous random variables, how to describe them
  - Expectation, variance, quantiles
- Applications through fundamental theorems
  - Law of large numbers, Central Limit Theorem
    $\rightarrow$ Estimate the mean, the variance
  - Knowing an estimate of the mean, the variance, estimate some probabilities

Outline

Three types of mathematical thinking

Terence Tao proposes that there are three types of mathematical thinking:

(Note that in course materials, this color is for hyperlinks; click above for his post)

- Pre-rigorous thinking
  - When I see a nail, I use a hammer
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  - Slower, but easier to detect and correct errors
- Post-rigorous thinking
  - No longer so dependent on step-by-step rules
  - Back to mainly using intuition, but now it's informed by rigor

The last two types are critical when you want to work on new problems!

- e.g. research, consulting, new applications/data types

Three types of mathematical thinking: an example

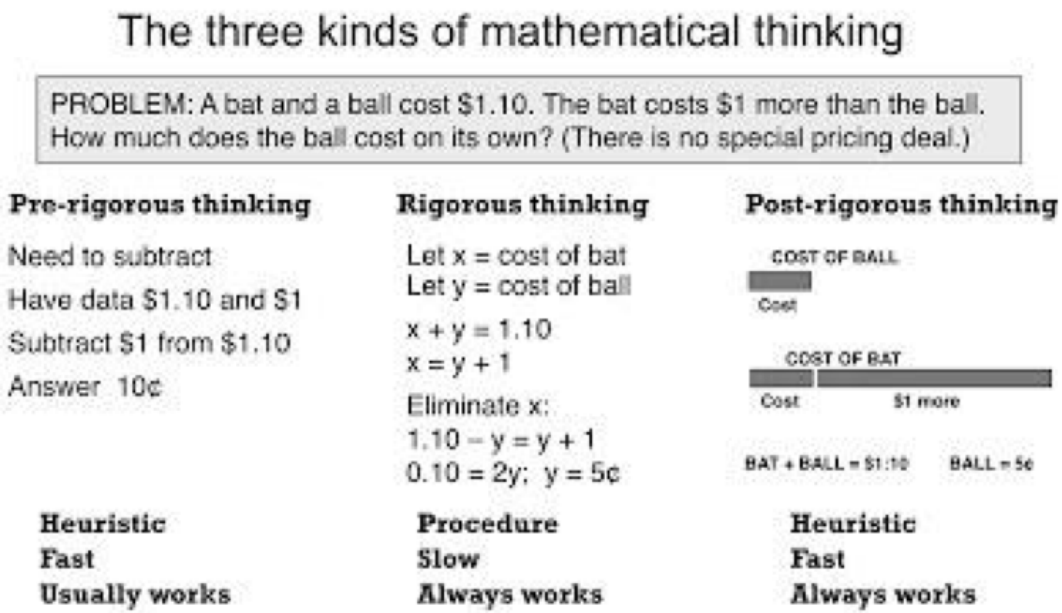

Image credit: MAA (www.mathvalues.org)

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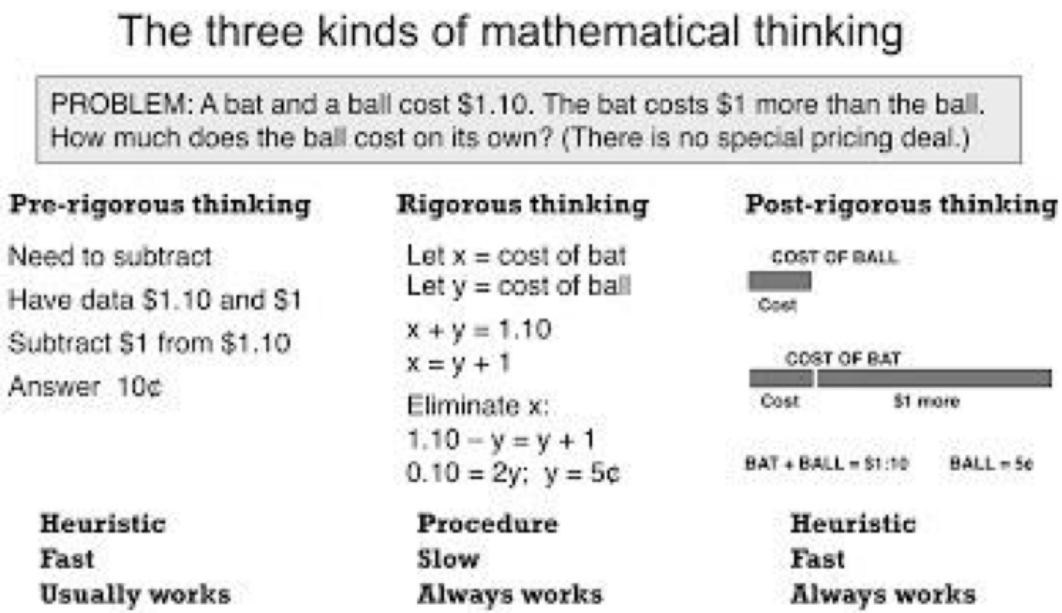

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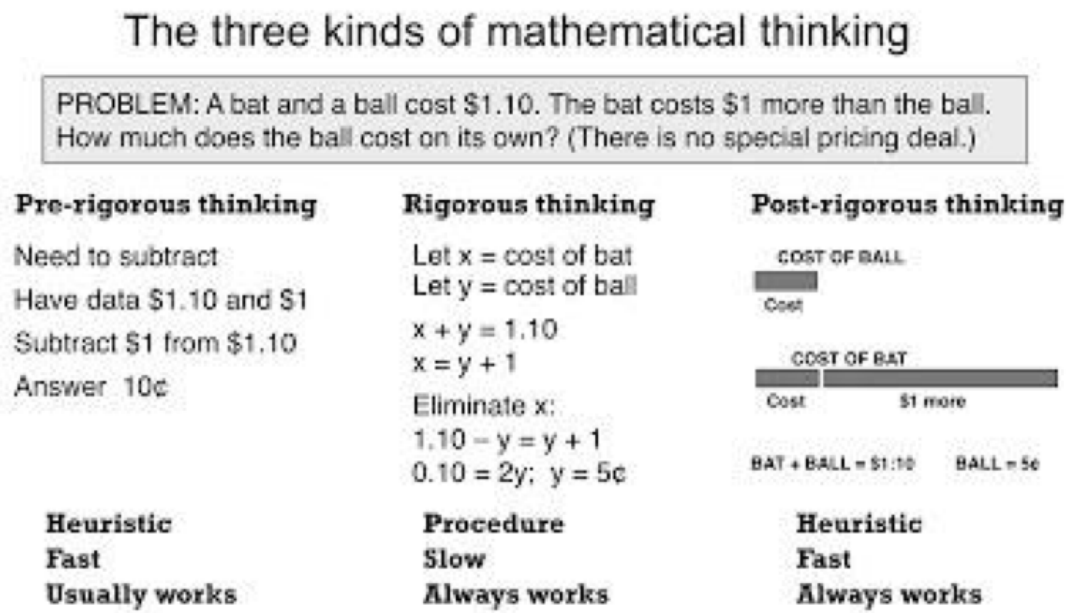
Takeaways:

- We need to practice rigor to make sure intuition is on a solid ground
  - Simple, well-defined examples are critical here!

Three types of mathematical thinking: an example



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Takeaways:

- We need to practice rigor to make sure intuition is on a solid ground
  - Simple, well-defined examples are critical here!
- But we want to keep trying to see the forest (intuition), not just the trees (rules)!