MATH/STAT 394, Homework 4

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Due Mon 18 July 2022

Remember to refer to the syllabus for homework instructions and guidelines.

Required exercises

Exercise 1. Let X be a random variable with density function

$$f_X(x) = \begin{cases} \frac{2}{9}x & \text{for } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability density function of $Y = X^2 - 3X$.

Exercise 2. Let X_i be the amount of money earned by a food truck on State Street on day i. From past experience, the owner of the cart knows that $\mathrm{E}[X_i] = \$5000$.

1. Use one of the concentration inequalities from class to provide an upper bound for the probability that the cart will earn at least \$7000 tomorrow.
2. Answer Part 1 again with the extra knowledge that $\mathrm{Var}[X_i] = 4500$.
3. Continue to assume that for all i we have $\mathrm{E}[X_i] = 5000$ and $\mathrm{Var}[X_i] = 4500$. Assuming that the amount earned on any given day is independent of the earning on other days, how many days does the cart have to be on State Street to ensure, with a probability at least 0.95, that the cart's average earnings would be between \$4950 and \$5050?

Exercise 3. My bus is scheduled to depart at 1pm. However, in reality, the departure time varies randomly, with an expected departure time of 1pm and a standard deviation of 6 minutes. Assume the departure time is normally distributed. If I get to the bus stop at 1:05, what is the probability that the bus has not yet departed?

Exercise 4. Let Y be a geometric random variable with parameter $p = 1/6$. For this problem you can use the textbook or the internet to look up the expectation and variance of a geometric random variable.

1. Use Markov's inequality to find an upper bound for $P(Y \geq 16)$.
2. Use Chebyshev's inequality to find an upper bound for $P(Y \geq 16)$.
3. Explicitly compute the probability $P(Y \geq 16)$ from the distribution of Y and compare with the upper bounds you derived.

Exercise 5. In a high school there are 1200 students. Estimate the probability that more than 130 students were born in January under each of the following assumptions. You do not have to use the continuity correction.

1. Assume that the months of the year are equally likely to be birthdays.

2. Assume instead that the days of the year are equally likely to be birthdays.

Exercise 6. A pollster would like to estimate the fraction p of people in a population who intend to vote for a particular candidate. How large must a random sample be in order to be at least 95% certain that the fraction $\hat{p}$ of positive answers in the sample is within 0.02 of the true p?

Exercise 7. We are throwing darts on a disk shaped board of radius 5. We assume that the position of the dart is a uniformly chosen point in the disk. The board has a disk shaped bullseye with radius 1. Suppose that we throw a dart 2000 times at the board, with each throw independent of the others. Estimate the probability that we hit the bullseye at least 100 times.

Extra credit

Exercise 8. You flip a fair coin 10,000 times. Approximate the probability that the difference between the number of heads and number of tails is at most 100.