MATH/STAT 394, Homework 2

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Due Fri 1 July 2022

Remember to refer to the syllabus for homework instructions and guidelines.

Required exercises

Exercise 1. Let's work a bit with independence:

1. Suppose that $P(A) = 1/3$, $P(B) = 1/3$, and $P(A \cap B^c) = 2/9$. Are A and B independent? Why or why not?

2. Show that if events A and B are independent, then A^c and B^c are also independent.

Exercise 2. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?

Exercise 3. Suppose we have 10 coins such that if the ith coin is flipped, heads will appear with probability $i/10, i = 1, \cdots, 10$. When one of the coins is randomly selected and flipped, it shows heads. What is the conditional probability that it was the fifth coin?

Exercise 4. A coin shows tails with probability p $(0 < p < 1)$ and head with probability $1 - p$. Flip this coin three times (the outcomes are independent). Let A denote the event that we have exactly one tails among the first 2 flips and B the event that we have exactly two tails among the last two flips.

1. Express $P(A), P(B), P(A \cap B)$ in function of p

2. For which value of p are events A and B independent?

Exercise 5. An urn contains 11 balls, 3 white, 3 red, and 5 blue balls. Take out two balls at random without replacement. You win \$1 for each red ball you select and lose \$1 for each white ball you select. Let X be the random variable that notes the amount you win. Find the probability mass function (pmf) of X.

Exercise 6. The probability that there is no accident at a certain busy intersection is 95% on any given day, independently of the other days.

1. Find the probability that there will be no accidents at this intersection during the next 7 days.

2. Find the probability that next October there will be exactly 2 days with accidents.

3. Today was accident free. Find the probability that there is no accident during the next 4 days, but there is at least one by the end of the 10th day.

Exercise 7. Two factories I and II produce phones for Awesome Phone Company (APC). Factory I produces 60% of all APC phones, and factory II produces the other 40%. 10% of the factory I phones and 20% of the factory II phones are defective. You purchase an APC phone and it is not defective. Let's assume this phone is uniformly randomly chosen from all APC phones. What is the probability that it came from factory II?

Extra credit

Exercise 8. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that ith hand has exactly one ace. Determine $p = P(E_1 E_2 E_3 E_4)$ by using the multiplication/factorization rule.