

# MATH/STAT 394, Homework 1

Jess Kunke

Due Mon 27 June 2022

Remember to refer to the syllabus for homework instructions and guidelines.

## Required exercises

**Exercise 1.** We roll a fair six-sided die twice.

1. Define the sample space  $\Omega$  and a probability  $P$  for this experiment that will allow you to solve the second part of this problem. Note that specifying a probability  $P$  means specifying the full map  $P : \Omega \rightarrow \mathbb{R}$ , or specifying the probability of each outcome.
2. Let  $A$  be the event that the second roll is larger than the first. Find  $P(A)$ .

**Exercise 2.** You have 3 green and 4 yellow balls in a bag, and you draw 3 balls one at a time without replacement.

1. Find the probability that the colors of the balls you draw are green first, then green again, then yellow.
2. Find the probability that the sample of 3 balls contains 2 green balls and 1 yellow ball.

**Exercise 3.** Suppose that a bag of scrabble tiles contains 5 Es, 4 As, 3 Ns and 2 Bs. I draw 4 tiles from the bag without replacement, then ignore the order in which I drew them. Assume that each time I draw a tile, all tiles in the bag have equal probability of being drawn. What is the probability that I get a total of 2 Es, 1 A and 1 N?

**Exercise 4.** From a group of 8 engineers and 6 urban planners, you form a committee consisting of 3 engineers and 3 urban planners. How many different committees are possible if

1. two of the engineers refuse to serve together?
2. one engineer and one urban planner refuse to serve together?

**Exercise 5.** Consider a standard set of 52 cards: there are four suits and 13 ranked face values (in order they are A, the numbers 2 through 10, J, Q, K), and each card is a unique combination of suit and face value. How many combinations of 5 cards give you

1. a flush (five cards of the same suit which are not in sequential order)? *Generally an ace A can be low or high, so both  $(A, 2, 3, 4, 5)$  and  $(10, J, Q, K, A)$  are considered sequential, but  $(J, Q, K, A, 2)$  is not.*
2. a full house? *This is three cards with the same face value, and two other cards that have the same face value as each other but different from the other three cards. For example, the 2's of three suits and the 5's of two suits make up a full house.*

**Exercise 6.** Let  $(X, Y)$  denote a uniformly chosen random point inside the unit square

$$[0, 1]^2 = [0, 1] \times [0, 1] = \{(x, y) : 0 \leq x, y \leq 1\}.$$

Let  $a$  and  $b$  be fixed numbers such that  $0 \leq a < b \leq 1$ . Find the probability  $P(a < X < b)$ , that is, the probability that the x-coordinate  $X$  of the chosen point lies in the interval  $(a, b)$ .

## Extra credit exercises

**Exercise 7.** Pick a random integer (uniformly) from 1 to 2000. What is the probability that the number you select is not divisible by any of the numbers 2, 3, or 5?

**Exercise 8.** Assume you are arranging 6 plants in a line; label the plants A, B, C, D, E, and F.

1. How many different arrangements are there?
2. If plants A and B must be adjacent, how many different arrangements are there?
3. If plants A and B must be adjacent and C and D cannot be adjacent, how many different arrangements are there?